MATHEMATICAL MODEL OF PREVENTIVE MAINTENANCE BASED ON COST MINIMIZATION

МАТЕМАТИЧЕСКАЯ МОДЕЛЬ ПРЕДУПРЕЖДАЮЩЕГО СОДЕРЖАНИЯ В ИСПРАВНОСТИ, ОСНОВАННОМ НА СНИЖЕНИИ ДО МИНИМУМА СТОИМОСТЕЙ

Conf.dr.ing. Vasiu T.1, Lect. Stoica D.2
Faculty of Engineering (at Hunedoara)1,2 – POLITEHNICA University of Timisoara

Abstract: In this study is presented a mathematical model of preventive maintenance which takes into account several stochastic factors that influence the failure rate and working life of an entity. It is assumed that preventive maintenance is done through imperfections: it is not reduced only the proper operation but the failure probability as well as much as the number of maintenance works is increased. The objective of this study is to determine the optimal diagram for planned maintenance works in order to reduce the related costs.

KEYWORDS: PREVENTIVE MAINTENANCE, CORRECTION FACTORS, OPTIMAL SOLUTIONS

1. Introduction

The condition for a successful preventive maintenance is to decide the proper moment of its execution. One of the most used concepts is periodical preventive maintenance, which specifies the interventions following to be done at equal time elements of continuous operation (e.g. 670 h for ball mills [4]). Another concept is the so-called sequential preventive maintenance, consisting in planned interventions done on unequal time elements of continuous operation. The first concept is more convenient, but sequential preventive maintenance is more realistic because takes into account that an entity should be recovered often with its ageing. The link between these two concepts is done by corrective maintenance, which is applied when the entity is operational shut-down. Corrective maintenance solves only the specific cause of failure and not all the status problems of the entity. By other words the corrective maintenance doesn’t change the failure rate and doesn’t increase the working life.

Two of the most used methods to determine the time elements of sequential preventive maintenance are based on maintenance works cost minimization and keeping the failure rate under an admitted limit, but just the first method makes the object of this study.

Nakagawa [2] adopts some correction factors for failure rates $h(t)$ and for the proper operation time elements $t$ within the preventive maintenance i.e.:

1. Failure rate during the next operational time element is $ah(t)$, where $h(t)$ refers to the previous time element, $a \geq 1$ is a correction factor and $t \geq 0$ represents the time effluxed from the previous intervention.
2. Time element $t$ for proper operation of the entity before preventive maintenance is reduced to $bt$ after intervention, where $b \leq 1$ is a proper operation reduction factor.

According to the proposal of Nakagawa, for a failure rate $h(t)$, $t \in (0, t_1)$, the preventive maintenance started at the moment $t_1$ determines a new failure rate $g(t)$, $t \in (t_1, t_2)$, which depends on the previous failure rate and maintenance work. This study proposes a specific form of $g(t)$, using both the concept of increasing the failure rate and reducing the proper operation according to the entity exploitation:

$$g(t_1 + x) = ah(t_1 + x)$$ (1)

Where $a \geq 1, 0 \leq b \leq 1$ and $x \in (0, t_2 - t_1)$. For $a = 1$ the proposed model is resized to the one of working life reduction, and $b = 0$ is equivalent to increasing the failure rate.

Using the proposed model is developed an optimal policy for preventive maintenance, with a major implication, i.e.: giving up the classic, non-economic approach of constant time elements of preventive maintenance execution;

2. Description of model and optimal solutions

At this moment is necessary to specify the notations that follow to be used:

$h(t)$ – failure rate;

$H(t)$ – cumulated failure rate;

$x_k$ – moments of preventive maintenance execution, $k = 1, 2, ..., N$;

$t_k = x_1 + x_2 + ... + x_k, k = 1, 2, ..., N$;

$y_k$ – continuous operation time immediately after the number „$k$” preventive maintenance, $k = 1, 2, ..., N$;

$N$ – number of proper operation time elements;

$a_k$ – failure rate correction factor after the number „$k$” preventive maintenance;

$1 = a_0 \leq a_1 \leq a_2 \leq ... \leq a_{N - 1}$;

$$A_k = \prod_{i=0}^{k-1} a_i, k = 1, 2, ..., N;$$

$b_k$ – correction factor of proper operation time;

$$b_0 \leq b_1 \leq b_2 \leq ... \leq b_{N - 1} \leq 1;$$

$c_m$ – corrective maintenance cost;

$c_p$ – preventive maintenance cost;

$c_e$ – overhaul cost (entity replacement);

$C$ – average cost of entity;

$$d_k = \left[ \frac{(1-b_k)^{a_k}}{A_k - A_{k+1}b_k^{a_k}} \right] \frac{1}{a_k - 1}$$

It is thought the situation in which an entity is subjected to the preventive maintenance at the moments $t_1, t_2, ..., t_{k+1}$ and overhauled or replaced at the moment $t_k$. Corrective maintenance is executed as consequences of failures appeared between the works of preventive maintenance. The overhaul from the moment $t_k$ makes the entity to be like a new one. The entity has the failure rate $A_kh(t)$ between the
number \( k\)-1” and \( k \) preventive maintenances, i.e. in the range \((t_{k-1} , t_k)\). The proper operation time is \(b_{k+1}y_{k+1}\) immediately after the number \( k\)-1” preventive maintenance, becoming \( y_k = x_k + b_{k+1}x_{k+1} + \ldots + b_{k+2}y_{k+2} - b_{k+1}x_1\) after the number \( k \)” preventive maintenance, meaning that the proper operation time is changed from \(b_{k+1}y_{k+1}\) la \( y_k \) in the range \((t_{k-1} , t_k)\). Obviously \( y_k = x_k + b_{k+1}y_{k+1}\) or \( x_k = y_k - b_{k+1}y_{k+1}\).

From [2], the entity average cost is:

\[
c_r + c_p(N - 1) + c_m \sum_{k=1}^{N} A_k [H(y_k) - H(b_{k-1}y_{k-1})]
\]

\[
C = \frac{\sum_{k=1}^{N-1} (l - b_{k}) y_{k} + y_{N}}{\sum_{k=1}^{N} (l - b_{k}) y_{k} + y_{N}}
\]

(2)

The next targeted object is to determine the proper operation time elements in order to minimize the average cost \( C \) [4].

In this model, the moments of time where the preventive maintenance done are \( t_1, t_2, ..., t_{k-1} \), with overhaul done at the moment \( t_N \). All values are considered as independent. By other words, the variables are \( N \) and \( y_k \) \((k = 1, 2, ..., N)\), meaning that the objective of this model is to determine the optimal values for \( N \) and \( y_k \) in order to minimize the average cost given by the relation (2).

According to [1] and relation (2), from equation \( \frac{\partial C}{\partial y_k} = 0 \) is acquired:

\[
A_k b(y_k) - A_{k+1} b_k h(b_k y_k) = A_N(l - b_k) h(y_N), \quad k = 1, 2, ..., N - 1
\]

and

\[
c_m A_h(y_N) = C
\]

(3)

Replacing each solution \( y_k \) of equation (3) in (4), we’ll acquire:

\[
A_N h(y_N) \left[ \sum_{k=1}^{N-1} (l - b_{k}) y_{k} + y_{N} \right] - \sum_{k=1}^{N} A_k [H(y_k) - H(b_{k-1}y_{k-1})] =
\]

\[
= \frac{c_r + c_p(N - 1)}{c_m}
\]

(5)

Where each \( y_k \) \((k = 1, 2, ..., N - 1)\) is a function of \( y_N \).

The algorithm to find the optimal maintenance time moments is:

1. Solving the equation (2) and finding the solutions \( y_k \) in accordance with \( y_N \);
2. Replacing the solutions of step 1 in equation (5), solving the equation and determination of \( y_N \);
3. It is chosen \( N \) to minimize \( A_N h(y_N) \), where \( y_N \) are the solutions acquired for step 2;
4. Calculation of \( y_k \) \((k = 1, 2, ..., N)\) through the relations of steps 1 and 2, for the value of \( N \) from step 3;
5. Calculation of \( x_k = y_k - b_{k+1}y_{k+1}, k = 1, 2, ..., N \).

3. Numerical example

This is thought that in situation of a ball mill [4] which for the failure rate is according to the Weibull distribution law:

\[
h(t) = \beta t^{\alpha - 1}
\]

Where \( \beta = 6.148 \times 10^9 \) and \( \alpha = 2.462 \).

According to the described model, equation (3) becomes:

\[
A_k b(y_k) - A_{k+1} b_k h(b_k y_k) = A_N(l - b_k) y_N = k, 1, 2, ..., N - 1
\]

Solving this equation is acquired:

\[
y_k = \left[ \frac{A_N(l - b_k)}{A_k - A_{k+1} b_k} \right]^{\frac{1}{\alpha}} y_N, \quad k = 1, 2, ..., N - 1
\]

(9)

Replacing the equation (9) in relation (4) is acquired:

\[
y_N = \left[ \frac{c_r + c_p(N - 1)}{c_m} \right]^{\frac{1}{\alpha}}
\]

(10)

Then \( A_N h(y_N) \) becomes:

\[
B(N) = \left[ \frac{c_r + c_p(N - 1)}{c_m} \right]^{\frac{1}{\alpha}} \sum_{k=1}^{N-1} d_k
\]

Minimization of \( B(N) \) is equivalent with minimization of the function:

\[
D(N) = \frac{c_r}{c_p} \quad \text{and} \quad D(N - 1) < \frac{c_r}{c_p}
\]

(11)

where \( D(N) = \frac{1}{A_N^{\alpha - 1}} \sum_{k=1}^{N-1} d_k \)

Optimal value \( N^* \) for \( N \) can be find solving the inequalities \( B(N + 1)^{\text{opt}} > B(N) \) and \( B(N) < B(N - 1) \). The two inequalities involve:

\[
\text{D}(N)^{\text{opt}} = \frac{c_r}{c_p} \quad \text{and} \quad \text{D}(N - 1)^{\text{opt}} < \frac{c_r}{c_p}
\]

The optimal operation time elements up to the preventive maintenance have the following form \( x_k = x_k - b_{k+1}y_{k+1}, k = 1, 2, ..., N \) where \( y_k \) are given through relation (9) and (10).

For an accurate calculus of values \( x_k \) \((k = 1, 2, ..., N)\) is necessary to be known the economic parameters \( c_m, c_p \), and \( c_r \), parameters \( \alpha \) and \( \beta \) of Weibull law and coefficients \( a_k \) and \( b_k \). For economic parameters is enough to know the ratios \( \frac{c_r}{c_p} \) and \( \frac{c_r}{c_m} \). Coefficients \( a_k \) and \( b_k \) are
recommended in [4]. In this numerical example \( c_r = 2, 5, 10, 20, 50, \)
\( c_p \)

\( \frac{c_r}{c_m} = 4, \) and \( a_k = \frac{6k + 1}{5k + 1}, b_k = \frac{k}{2k + 1}, k = 0, 1, 2,... \)

The algorithm solving has been done using the MathCAD [3] software, and the results of simulation are given in Table 1.

Table 1. Cycles proposed for ball mill preventive maintenance

<table>
<thead>
<tr>
<th>( c_r/c_m )</th>
<th>2</th>
<th>5</th>
<th>10</th>
<th>20</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N )</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

| Continuous operation time elements between two planned interventions [hours] |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| \( x_1 \)       | 1464.889        | 2341.618        | 2952.949        | 3691.936        | 5057.92 |
| \( x_2 \)       | 1249.767        | 1576.046        | 1970.458        | 2699.51         |
| \( x_3 \)       | 972.58          | 1226.493        | 1533.428        | 21007.83        |
| \( x_4 \)       | 1321.36         | 1021.004        | 1276.514        | 1748.813        |
| \( x_5 \)       | 871.455         | 1089.54         | 1492.661        |                 |
| \( x_6 \)       | 752.843         | 941.245         | 1289.497        |                 |
| \( x_7 \)       | 1119.014        | 818.629         | 1121.515        |                 |
| \( x_8 \)       | 714.931         | 979.45          |                 |                 |
| \( x_9 \)       | 626.059         | 857.695         |                 |                 |
| \( x_{10} \)    | 956.946         | 752.473         |                 |                 |
| \( x_{11} \)    |                | 661.037         |                 |                 |
| \( x_{12} \)    |                | 581.281         |                 |                 |
| \( x_{13} \)    |                | 511.526         |                 |                 |
| \( x_{14} \)    |                | 450.399         |                 |                 |
| \( x_{15} \)    |                | 701.93          |                 |                 |

5. References


4. Conclusions

This study presents an execution method of preventive maintenance starting from the observation in accordance with repairs don’t reduce only the working life of an entity but also change the failure rate. As example has been chosen a ball mill whose distribution law is Weibull one, because the algorithm described in this study can not be solved as for a general case.

From table 1 is ascertained that operation time elements between two successive repairs are reduced, except the latest time element for which is observed a certain increase. That means that it is properly to be done a preventive maintenance in accordance with entity age and, in the same time, it is recommended that latest planned intervention to be executed as late as possible, because the next repair work is the overhaul one.