Abstract: This paper describes the vibroacoustic coupling between the structural vibrations and internal sound fields of thin structures. In this study, a cylindrical structure with thin end plates is subjected to the harmonic point force at one end plate or both end plates and a natural frequency of the end plates is selected as the forcing frequency. The resulting vibroacoustic coupling is then analyzed by theoretically and experimentally by considering the dynamic behavior of the plates and the acoustic characteristics of the internal sound field as a function of the cylinder length. The length and phase difference between the plate vibrations, which maximize the sound pressure level inside the cavity, are clarified theoretically. The theoretical results are validated experimentally through an excitation experiment using an experimental apparatus that emulates the analytical model. Moreover, the electricity generation experiment verifies that sufficient vibroacoustic coupling can be created for the adopted electricity generating system to be effective as an electric energy-harvesting device.

Keywords: VIBROACOUSTIC COUPLING, CYLINDRICAL STRUCTURAL, ENERGY HARVESTING, SOUND FIELD INTERNAL, PHASE DIFFERENCE

1. Introduction

Recently, scavenging ambient vibration energy and converting it into usable electric energy via piezoelectric materials has attracted considerable attention. Typical energy harvesters adopt a simple cantilever configuration to generate electric energy via piezoelectric materials, which are attached to or embedded in vibrational elements. High-amplitude excitations reduce the fatigue life of these harvesters. Thus, placing appropriate constraints on the amplitudes is one of significant ways to improve the performance of harvesters. A cantilever beam, whose deflection was constrained by a bump stop, was modeled. The effect of electromechanical coupling was estimated in a parametric study, where the placement of the bump stop and the gap between the beam and stop were chosen as parameters.

In an attempt to control noise in an airplane, an analytical model for investigating coupling between the sound field in an aircraft cabin and the vibrations of the rear pressure bulkhead was proposed. A cylindrical structure adopted as the analytical model, in which the rear pressure bulkhead at one end of the cylinder was assumed to be a circular plate, was examined under various conditions. The plate was supported at its edges by springs whose stiffnesses could be adjusted to simulate the various support conditions. These investigations clarified the influence of the support conditions on the sound pressure of an internal sound field coupled with the vibration of the end plate.

To develop a new electricity generation system, we adopt an analytical model similar to the above-mentioned cylindrical structure with plates at both ends, because the vibration area of the model on which piezoelectric elements can be installed is twice as large as that in case of a single plate. The cylinder length is varied over a wide range, while the harmonic point force is applied to one end plate or both end plates and its frequency is selected to cause the plate to vibrate in a fundamental mode. Vibroacoustic coupling that occurs between plate vibrations and the sound field in the cavity is investigated theoretically and experimentally in terms of the vibration and acoustic characteristics. In the experiment, the acceleration of the plate vibrations, the phase difference between them and the sound pressure level inside the cavity are considered as significant characteristics of the plate vibrations and sound field. These experimental results demonstrate the underlying theoretical analysis based on this model, as well as the conditions that maximize the vibration and sound pressure levels. Furthermore, the effect of vibroacoustic coupling is estimated from an electricity generation experiment performed with piezoelectric elements.
3. Experimental apparatus and method

Figure 2(a) shows the experimental apparatus used in this study. The structure consists of a steel cylinder with circular aluminum end plates that are 3 mm thick. The cylinder has an inner radius of 153 mm, and this length can be varied from 500 to 2000 mm to emulate the analytical model. One end plate or both end plates are subjected to the point force, whose frequency makes the plate excite in the (0,0) mode. In case of the harmonic excitation of both ends, these forces are applied to the respective plates via small vibrators, and their amplitudes are controlled to be 1N. The positions of the point forces $r_1$ and $r_2$ are normalized by radius $a$ and are set to $r_1/a = r_2/a = 0.4$. In the excitation experiment, the main characteristic is the phase difference between the plate vibrations. Therefore, acceleration sensors are installed on both plates to measure this phase difference. To estimate the internal acoustic characteristics, the sound pressure level in the cavity is measured using condenser microphones with a probe tube. The tips of the probe tubes are located near the plates and the cylinder wall, which are the approximate locations of the maximum sound pressure level when the sound field becomes resonant.

The electricity generation experiment is performed with piezoelectric elements installed at each center of both plates. The electric power generated by the expansion and contraction of the piezoelectric elements is discharged through the resistance circuit. To grasp the effect of vibroacoustic coupling on energy-harvesting, the electric power is measured with and without the cylinder and is estimated by the comparison of both cases. In case of the estimation, the electric power is normalized by the vibration power supplied with the plate, which is obtained from the point force and flexural displacement.

4. Results and discussion

4.1 Vibroacoustic coupling between plate vibration and sound field

In the theoretical study, the plates are assumed to be aluminum having a Young’s modulus $E$ of 71 GPa and a Poisson’s ratio $\nu$ of 0.33. The radius $a$ and thickness $h$ of the plates are constant at 153 mm and 3 mm, respectively, whereas the length of the cylindrical sound field having the same radius as that of the plates varies from 100 to 2000 mm. The support conditions of the plates, which have flexural rigidity $D = Eh^3/12(1-\nu^2)$, are expressed by the non-dimensional stiffness parameters $T_p (= T_1a^4/D = T_2a^4/D)$ and $R_n (= R_1a^2/D = R_2a^2/D)$. These values are identical for both plates. If $R_n$ ranges from 0 to $10^5$ when $T_p$ is $10^3$, the support condition can be assumed from a simple support to a clamped support. The actual condition adopts $T_p = 10^3$, $R_n = 10^4$ to get closer to the experimental support condition. These plate 1 and 2 are subjected to the point forces $F_1$ and $F_2$ which are set to 1 N and are located at $r_1/a = r_2/a = 0.4$ respectively, as well as the actual excitation experiment. In particular, the analysis in which only one end plate is excited is carried out with taking $F_2$ as 0 N.

The plate and sound field eigenfrequency characteristics involved in the vibroacoustic coupling are represented by the natural frequency $f_{nm}$ corresponding to the $(n,m)$ mode and the resonance frequency $f_{nm}$ corresponding to the $(n,p,q)$ mode. The excitation frequency is chosen as $f_{00}$ that makes the plates vibrate in the (0,0) mode. Although the phase $\phi$ of plate 1 is fixed at 0 deg, the phase $\phi$ of plate 2 ranges from 0 to 180 deg, and then they are related by the phase difference $\phi$ as follows:

$$\phi = \phi_2 - \phi_1.$$  \hspace{1cm} (1)

Figure 3 shows the variations in $L_{pv}$ with $L$, when only plate 1 is excited and $\phi$ is set to 0, 10 and 90 deg. $L_{pv}$ varies only slightly over the entire range of $L$ when $\phi = 0$ deg, but varies substantially and exhibits peaks near $L = 610, 1220$ and $1830$ mm when $\phi = 10$ and 90 deg. The values of $L_{pv}$ are suppressed in the range except the vicinities of their lengths when $\phi = 10$ and 90 deg and is almost identical at all phase differences near $L = 460, 920$ and 1560 mm. This change in $L_{pv}$ with $\phi$ indicates that the acoustic characteristics depend strongly on the vibration of plate 2, i.e. there are ranges of $\phi$ that intensify or suppress coupling between the plate vibrations and sound field. Here the values of $\phi$ at which $L_{pv}$ is a maximum are denoted by $\phi_{max}$.

In Fig. 4, the variations in $\phi_{max}$ with $L$ and the values of $\phi_{max}$ that maximize $L_{pv}$ are plotted by a line and circles, respectively. $\phi_{max}$ is approximately $87$ deg at $L = 100$ mm and decreases gradually with increasing $L$ up to approximately $L = 460$ mm, where $\phi_{max}$ suddenly increases to over 90 deg and then decreases again with increasing $L$. This behavior of $\phi_{max}$ is repeated in a similar manner as $L$ increases to $L = 2000$ mm. Peaks in $L_{pv}$ indicate that vibroacoustic coupling between the plate vibrations and sound field is promoted at approximately 90 deg. $f_{nm}$ and $f_{nm}$ must be approximately equal for the promotion of this coupling, so that the acoustic modes involved in vibroacoustic coupling are greatly influential around the lengths at which $L_{pv}$ peaks. Figure 5 shows the distributions of the sound pressure level $L_{pv}$ averaged over a lateral cross-sectional plane $(rt$-plane) through the cavity along the $z$ direction as (1) $L = 1220$ mm at $\phi_{max} = 90$ deg, (2) $L = 900$ mm at $\phi_{max} = 66$ deg and (3) $L = 910$ mm at $\phi_{max} = 152$ deg. Since $L_{pv}$ peaks and the sound field becomes resonant at $L = 1220$ mm, the $L_{pv}$ distribution represents clearly the longitudinal order $q = 2$. The shift in the acoustic mode causes $\phi_{max}$ to change abruptly, taking place between $q = 1$ and 2 at $L = 900$ and 910 mm. These distributions correspond to (1), (2) and (3) in Fig. 5, in which the range of the dominant acoustic mode is classified in color. The sound fields, which are classified in the ranges of 470 to 900 mm, 910 to 1560 mm and 1570 to 2000 mm, are dominated by the (0,0,1), (0,0,2) and (0,0,3) modes, respectively.

Figure 6 shows the sound pressure levels $L_{p1}$ and $L_{p2}$, which are measured near plates 1 and 2, respectively, as functions of $L$. The theoretical level $L_{pnm}$, which is maximized at each $L$ when the phase difference $\phi$ ranges from 0 to 180 deg, is also indicated to compare with the experimental results. $L_{p1}$ and $L_{p2}$ show peaks around 615, 1275 and 1900 mm, and these levels are almost coincident for each peak. However, they decrease in the middle range of those lengths. In particular, decreases in $L_{p1}$ are remarkable and their differences expand.
Figure 5 shows \( \phi_{\text{max}1} \), \( \phi_{\text{max}2} \), \( \phi_{\text{min}1} \) and \( \phi_{\text{min}2} \) as functions of \( L \). \( \phi_{\text{max}1} \) is constant at 180 deg for \( L \) ranging from 100 to 390 mm and decreases abruptly up to 0 deg at \( L = 400 \) mm. Then, \( \phi_{\text{max}1} \) remains constant at 0 deg up to \( L = 610 \) mm, increasing gradually with \( L \), and returns to 180 deg at \( L = 1220 \) mm, increasing somewhat abruptly near \( L = 970 \) mm. Beyond \( L = 1220 \) mm, \( \phi_{\text{max}1} \) is again constant at 180 deg up to \( L = 1570 \) mm, and this behavior is repeated as \( L \) increases to \( L = 2000 \) mm. \( \phi_{\text{min}2} \) exhibits gradual and abrupt changes similar but alternate to \( \phi_{\text{max}1} \). For example, when \( L \) increases, a gradual decrease occurs in \( \phi_{\text{max}2} \) between \( L = 100 \) and 620 mm and an abrupt increase occurs near \( L = 970 \) mm. Both \( \phi_{\text{max}1} \) and \( \phi_{\text{max}2} \) shift between 0 and 180 deg with changing somewhat abruptly near \( L = 970 \) mm. Beyond \( L = 1220 \) mm, \( \phi_{\text{max}1} \) and \( \phi_{\text{min}2} \) shift between 0 and 180 deg with changing \( L \) and intersect at approximately 90 deg and near the length at which \( L_{\text{pp}} \) peaked in Fig. 7. \( \phi_{\text{min}1} \) and \( \phi_{\text{min}2} \) behave exactly alike but opposite to \( \phi_{\text{max}1} \) and \( \phi_{\text{max}2} \).

In Fig. 7, the theoretical results for \( \phi_{\text{max}} \), where \( L_{\text{pp}} \) peaks and the experimental results for \( \phi_{\text{exp}} \) as \( L_{\text{pp}1} \) and \( L_{\text{pp}2} \) are maximized at each \( L \) are also plotted. \( \phi_{\text{exp}} \) ranges greatly between in-phase and out-of-phase and \( \phi_{\text{max}} \) exists in the process where \( \phi_{\text{exp}} \) changes abruptly. Then, \( \phi_{\text{exp}} \) lies in the light yellow areas surrounded by \( \phi_{\text{max}1} \) and \( \phi_{\text{max}2} \) in the \( L \) ranges longer than the lengths when the sound pressure level peaks and occurs in the yellowish green areas surrounded by \( \phi_{\text{min}1} \) and \( \phi_{\text{min}2} \) on the other side. In other words, since vibroacoustic coupling is gradually weakened with increasing \( L \) after the peaks of \( L_{\text{pp}1} \) and \( L_{\text{pp}2} \), the acoustic mode involved in coupling shifts to that having the next order \( q \).

Figure 8 shows the vibration levels \( L_{\text{v}} \) and \( L_{\text{a}} \) of plates 1 and 2 as functions of \( L \) and the accelerations \( \alpha_1 \) and \( \alpha_2 \) of plates 1 and 2 are also plotted to compare with the theoretical plate behavior. \( L_{\text{v}1} \) is smaller than \( L_{\text{v}2} \) in the ranges of 100 to 610 mm, 800 to 1200 mm and 1300 to 1820 mm, so that \( L_{\text{v}1} \) and \( L_{\text{v}2} \) intersect at a number of \( L \) and the intersections take place around the lengths where \( L_{\text{pp}} \) peaks. The actual motion of plate 1 is almost suppressed by that of the vibrator since plate 1 is supported by the vibrator; hence, \( \alpha_1 \) is approximately constant in the entire \( L \) range. However, since the motion of plate 2 depends greatly on the behavior of the sound field, that is, the only excitation source for plate 2, \( \alpha_2 \) peaks at \( L = 615 \), 1275 and 1900 mm and is suppressed in the other range of the lengths, such as variations in \( L_{\text{pp}1} \) and \( L_{\text{pp}2} \). As a result, the relative relationship of \( \alpha_1 \) and \( \alpha_2 \) becomes opposite to that of \( L_{\text{v}1} \) and \( L_{\text{v}2} \) as shown in the colored regions. This is because the experimental model cannot completely emulate the theoretical model in the flexural displacement.

To grasp the effect of the excitation method on vibroacoustic coupling, the excitation condition in which both plates are subjected to the same excitation force is taken. Figure 9 shows the variations in \( L_{\text{pp}} \) corresponding to \( \phi_{\text{max}} \) in the analysis and the variations in the sound pressure level \( L_{\text{p}} \) that is measured in the experiment and is maximized when the phase difference between both point forces ranges from 0 deg to 180 deg. Peaks in \( L_{\text{pp}} \) appear at \( L = 610 \), 1230 and 1840 mm. These peaks are known to be caused by the \((0,0)\), \((0,1)\), \((0,2)\) and \((0,3)\) modes, respectively. Note that \( L_{\text{p}} \) increases greatly at 625, 1250 and 1850 mm and peaks at values similar to the peaks in \( L_{\text{pp}} \). However, \( L_{\text{p}1} \) and \( L_{\text{p}2} \) is hardly distinguished in the middle range of lengths where the sound pressure levels peak, having been different in the results for the excitation of one end plate, as shown in Fig. 6.

Figure 10 shows that the experimental phase difference \( \phi_{\text{exp}} \), at which \( L_{\text{p}} \) is maximized, is compared with the theoretical phase difference \( \phi_{\text{max1}}, \phi_{\text{max2}}, \phi_{\text{min1}} \) and \( \phi_{\text{min2}} \) as functions of \( L \) as well as Fig. 7. The shifts in the \((0,q)\) modes are also represented by the changing colors. \( \phi_{\text{exp}} \) shifts between 0 and 180 deg with changing \( L \) and corresponds approximately with the \( \phi_{\text{max1}} \) and \( \phi_{\text{min2}} \) that behave uniformly in the vicinity of the in-phase side or the out-of-phase side. In this way, the behavior of the phase difference is very different in the excitation method.
4.2 Electricity generation characteristics

In this section, we consider electricity generation by the plate vibrations coupled with the sound field. In this case, the electricity generation is estimated by the comparison between the electric power \( P_e \) via the piezoelectric element and the mechanical power \( P_m \) supplied to the plate by the vibrator that is obtained from the relationship between the point force and flexural displacement at the excitation point. Here, \( P_m \) divided by \( P_e \) is denoted as \( P_{em} \), which is considered as energy-harvesting efficiency.

Figure 11 shows variations in \( P_{em} \) with \( L \); the powers are measured in the experimental apparatus via the cylinder shown in Fig. 2 when one end plate is excited by the point force. Although \( P_{em} \) of plate 1 remains almost constant in the entire range of \( L \), \( P_{em} \) of plate 2 increases greatly at \( L = 615, 1275 \) and 1900 mm. It is natural that the relationship between \( P_{em} \) of plate 1 and 2 is derived from the behavior of \( \alpha_1 \) and \( \alpha_2 \) in Fig. 8. Moreover, to grasp the effect of vibroacoustic coupling on electricity generation, although \( P_e \) is taken as the power ratio of \( P_{em} \) measured with and without the cylinder and is considered as the effect of vibroacoustic coupling on energy-harvesting, \( P_{em} \) measured with the cylinder is the total value with respect to plates 1 and 2. Figure 12 shows variations in \( P_e \) with \( L \); the results for the excitation of both plates are also indicated to study the effect of the excitation method. \( P_e \) for the excitation of one end plate is maximized as the sound pressure level peaks due to the promotion of vibroacoustic coupling. This is because the plate vibration on the non-excited side contributes strongly to the electricity generation, whereas their \( L \) ranges are limited to the narrow regions in comparison with the sound pressure level. The remarkable result is that the acoustic energy from the sound radiation can be harvested through vibroacoustic coupling.

5. Conclusion

To apply vibroacoustic coupling to electricity generation, as a means of harvesting energy from vibration systems, coupling between plate vibrations and a sound field was investigated theoretically and experimentally for a cylindrical structure with thin circular end plates. The end plate was subjected to a harmonic point force. Moreover, the effect of vibroacoustic coupling on the harvest of energy was estimated from the electricity generating experiment. The present study focused on promoting the vibroacoustic coupling to increase the flexural displacements of the plates and the sound pressure level inside the cavity.

As a result of the estimation of vibroacoustic coupling from various viewpoints, the theoretical study reveals that the closeness of eigenfrequencies and the similarity of modal shapes between the plate vibrations and sound field are indispensable for promoting coupling. The experimental results confirm the theoretical estimation of increasing the flexural displacement and sound pressure level via the promotion of vibroacoustic coupling and support the complicated acoustic characteristics deduced from the theoretical results. In particular, changes in the cylinder length shift the acoustic mode in the longitudinal order and vary periodically the phase difference between both plate vibrations. It is validated that the phase difference is greatly different in the excitation method. When vibroacoustic coupling is promoted, the electricity generation experiment verifies that the promotion of coupling causes the generation efficiency to improve in comparison with the electricity generation caused only by the plate vibration without coupling.

6. References