INTRODUCTION

The implementation of many technological processes, as well as the work of modern thermal power and heat generating plants in many cases is related to the release of the large amount of energy in the form of heat into the environment. Therefore, considerable interest in new sources of energy, as well as new types of heat engines and power converters arises during recent years in the energy sector. In particular, a renewed interest in engines with an external supply of heat (EESH). These engines convert thermal energy of the different sources of medium and high temperature potential (600 ... 1500 °K) into mechanical energy.

EESH are typically used in stationary systems – small scale energy, local systems of heat and cooler supply, autonomous power generators, etc. The most used EESH are the Stirling cycle engines. In these engines, the main supply of heat to the working body occurs from the outside (in the heater) and heat removal – in the cooler. The working body of the engine is located in a closed inner loop of variable volume, over which particular thermodynamic cycle takes place. The possibility of local and alternative fuels as well as production wastes usage for such engines allows reducing the problem of energy supply, reducing the consumption of expensive fuels and, in many cases, reduce harmful emissions into the environment. EESH based on Stirling engines have several advantages over internal combustion engines (ICE): the working body circulates in a closed volume, it is theoretically possible to achieve high efficiency, minimal harmful impurities in the exhaust gases, the possibility of usage of a heat source of any kind, the absence of a gas distribution valves in the main body of the engine, absence of periodic micro explosions in the working cylinders, low noise level. As the weaknesses of the Stirling engine it is necessary to note the following: increased dimensions of the engine cooling system compared to the ICE, a significant loss of heat at the heat exchangers in one unit and the presence of reversing non-stationary flows of the working body, high average pressure in the cylinder with power of more than 40 kW.

Stirling engine is more complicated than conventional heat engines. Manufacturing costs (including regenerators and heat exchangers, etc.) are higher than that of the ICE, however, operation costs are much lower. Currently, number of companies are performing intensive researches in order to develop less expensive modifications of the Stirling engine adapted for the implementation in autonomous cogeneration systems utilizing the heat of combustion of domestic thermal generation plants.

The thermodynamic cycle of the modified β-type Stirling engine with unidirectional pulsating motion of the working body in recuperative heater and cooler has also been proved. The characteristic features of thermodynamic cycle of the engine compared to the classical thermodynamic Stirling cycle have been analyzed as well. The proposed simplified version of the Stirling engine with relatively low-power can be used in a cogeneration system for heat recovery of combustion products in a domestic flame furnaces in order to produce mechanical energy and then convert it, e.g. into electricity.

KEYWORDS: STIRLING ENGINE, RECUPERATIVE HEAT EXCHANGER, THERMODYNAMIC CYCLE, REGENERATOR, PULSATING MOTION COGENERATION SYSTEM

1. Introduction

2. Prerequisites and means for solving the problem
3. Means for Solving the Problem

Theoretical thermodynamic cycle of the classical Stirling engine with an ideal regenerator, provided the reversibility of all processes and the invariability of physical properties of working body consists of two isotherms and two isochores Fig.1. It describes four consecutive processes:

- isothermal compression 1-2 of the working body in the cold chamber at the temperature of \( T_1 \) from the volume \( V_1 \) to volume \( V_2 \), (compression ratio of the working body \( e = V_1/V_2 \), heat from the working body is removed in the cooler – environment);
- isochoric process 2-3-4 of the working body movement from the cold chamber to the hot one through regenerator with the increase of the body temperature from \( T_5 \) to \( T_6 \) (the heat from the working body is transferred to the regenerator nozzle), and further heating of the body to \( T_4 \) in the preheater (external heat source);
- isothermal expansion 4-5 of the working body in the hot chamber at the temperature of \( T_4 \) (the heat from the external source with the temperature of \( T_4 \) is transferred to the working body);
- isochoric process 5-6-1 of the working body movement to the cold area through regenerator with cooling from \( T_3 \) to \( T_6 \) (the heat from the working body is transferred to the regenerator nozzle) and subsequent cooling of body in the cooler to the temperature \( T_1 \).

\[
\frac{T_4 - T_1}{T_1 + \frac{T_6 - T_1}{(\gamma - 1)/\gamma}}(1 - \beta) = \frac{T_6 - T_1}{T_1 + \frac{T_6 - T_1}{(\gamma - 1)/\gamma}}(1 - \beta)
\]

Fig.1. Ideal cycle of the Stirling engine, \( T^* \) and \( T_s \) is the steady-state temperatures.

All thermo- and gas-dynamic processes in the Stirling engine are non-stationary. During one cycle the flow rate, temperature, pressure and density of the working body vary periodically. In general, the thermal efficiency \( \eta_t \) of the Stirling engine including efficiency of the heat exchanger-regenerator \( 0 < \beta < 1 \) is given by the formula [1-5]:

\[
\eta_t = \frac{T_6 - T_1}{T_1 + \frac{T_6 - T_1}{\gamma - 1}}(1 - \beta)
\]

Here: \( \delta = V_1/V_2 \) is combustion ratio of the working body, \( \gamma \) is adiabatic exponent.

Let us note that in refs. [2, 3 and 5], the coefficient \( \beta \) is defined differently. In refs. [2, 3] the value of \( 0 < \beta < 1 \) is given on the basis of experimental data. In ref. [5] theoretical method for determining the value of \( \beta \) is proposed, taking into account the cyclic shift of the regenerator from "heater" to "cooler" states for a finite number of cycles \( n = 1,2,3 \). It is shown that in the extreme case of steady periodic process with \( n \rightarrow \infty \), the maximum possible value of the regeneration ratio is \( \beta = 0.5 \). It corresponds to the ideal heat transfer between the working body and the regenerator with infinite mass.

The maximum value of the thermal efficiency of an ideal Stirling cycle is determined by formula (1), (1-\( \beta \))=0.5. The case when \( \beta = 0 \) corresponds to the absence of the regenerator.

In the absence of the regenerator isochoric heating process 2–4 theoretically should occur in regenerative heater of the appropriate construction and isochoric cooling process 5–1 in regenerative cooler. It is necessary to note that practical solution in this case may not be simple, as the absence of the regenerator – heat-storage assumes refusal from the reciprocating movement of the working body in it and transfer to the unidirectional movement of the working body in the heat exchangers. Changing flow conditions may significantly affect the value of the hydraulic resistance in the internal hydraulic engine networks, as well as to change the non-stationary pattern of thermal loads of cold and hot chambers of the cylinder. The practical solution of the problem of the working body unidirectional movement may be accomplished in different ways.

4. Solution of the problem and results

In this paper, this problem is solved by installing the breaker valve at the output of the recuperative cooler of the working body or at the inlet to the heater located in the hot upper part of the cylinder. The principle of operation of such modified Stirling \( \beta \)-type engine is described in refs. [6,7].

For each cycle the flow of the working body subsequently and separately moves in the heat exchangers in one direction in the proposed engine design [6]. In this case, it is heated only in the heater and in the cooler it is only cooled. As a result established temperatures \( T^* \) and \( T' \) (Fig. 1) will be closer, respectively, to \( T_2 \) and \( T_4 \) and the effectiveness of heat exchanger will be higher as there will be no reciprocal movement of the working body, which defines periodic change of cogeneration modes of the heat exchanger performance ("cooler" to "heater" and vice versa).

In the engine cooler water, which has good thermal properties is used as coolant, allowing to obtain high heat transfer coefficients at a moderate temperature difference of the wall and fluid. Let us note that in this scheme, cooler serves as a connecting compression channel between the lower (cold) and the upper (hot) chambers of the cylinder at the same time to move the working body from the cooler to the heating zone of the displacer piston.

Heat exchanger – recuperative heater, the heating surfaces of which are arranged in the hot flow of exhaust gases of the heating furnace is also connecting expansion channel through which the working body moves from the hot chamber to the area between the pistons and then to the cooler. Unidirectional movement of the working body in reciprocal movement of the pistons is provided by installing the breaker valve at the outlet of the heat exchanger – cooler.

Breaker valve is closed at the cooling stage of the working body during the filling of the cooler and connecting channels as well as at the subsequent stage of the isothermal compression of the working body by the working piston that moves downwards under the effect of an external force.

As in the case of the classical Stirling cycle it is assumed that isothermal process is supported by the corresponding heat removal in the cooler. This condition is chosen to simplify comparison with the classical cycle and in the general case may be replaced by universal polytropic process. Earlier in [8] it was shown that for the considered engines with relatively low power at air pressures lower than 10 bar, air consumptions \( \sim 0.0003 \text{m}^3/\text{s} \) and pulsation frequencies \( \sim 10 /\text{s} \), it is possible practically neglect the pressure losses during the air flow through the recuperative cooler.

Taking into account this condition curve 1-2 in Fig. 1 may be represented as two segments (Fig. 2) for the studied engine modifications: isobars 6-1 (pressure \( P_6 = P_1 \)) and isotherm 1-2 (\( T_1 = T_2 = \text{const} \)).
cooler), breaker valve, and unidirectional movement of the working body in the heat exchangers. In contrast to the classical Stirling cycle shown in Fig. 2 the thermodynamic cycle of the modified engine with recuperative heat exchangers (heater and cylinder is performed by displacer piston. In the ideal cycle without cooler (isotherm temperature 6-3) is higher than the temperature T₀ of the cold coolant of the ideal recuperative cooler at the point 1 (in the ideal cooler in the absence of losses it is assumed that T₀=T₆). In this case, the isotherm 1-2 is the part of the ideal 1-2, according to which the heat is removed in the ideal Stirling cycle at the cold coolant temperature T₀ (theoretically possible lower limit of the operating temperatures of the cycle). For cogeneration plants with Stirling engine this threshold plays an important role. Positive result work performed by the mixed cycle, in p–v-coordinates is defined by the area 124561. Dashed area 1236 defines the increase of the resulting useful work of the cycle in comparison to the classical cycle (3-4-5-6) without breaker valve with the same minimum temperature of T₆. From Fig. 2 it follows that the useful work increases with the degree of preliminary isobaric compression of the working body εₚ₆ in the cooler (process 6-1, εₚ₆ = v₀/v₁ = p₀/p₆ = T₅/T₁ at p₆ = p₁) and the associated pressure increase ratio λ = p₆/p₂ of the isochoric heat supply increase during 2-3 at v₂=const.

In general, the main characteristics of the considered mixed cycle are: isothermal compression ratio εₜ = v₂/v₁, isobaric compression ratio εₚ₆ = v₀/v₁, temperature increase ratio τ = T₅/T₁, (T₅ = T₆, T₁ = T₂ ≤ T₀), isothermal expansion ratio δ = v₅/v₄, pressure increase ratio λ = p₆/p₂, specific heat capacity ratio cₙ₂/cₙ₁ = k.

In the general case the thermal efficiency of the cycle is [4]

\[ η = (\sum q_i - \sum q_i')/\sum q_i = 1 - (\sum q_i')/\sum q_i. \]  

(2)

where each term corresponds to the total specific heat qᵢ transferred to the working body (index 1) or removed from the working body (index 2). Taking into account Fig. 2, and the usual standard assumptions made in the analysis of the thermodynamic cycle of the Stirling engine [4], it is possible to write

\[ \sum q_i = q₁ + q₄' + q₅' = RT\ln δ + c₁(T₄ - T₁) + c₁(T₄ - T₅) + c₂(T₃ - T₂). \]  

(3)

\[ \sum q_i = q₂ + q₄' + q₅' = c₃(T₃ - T₀) + c₆(T₆ - T₀) + RT\ln δ, \]  

(4)

\[ η = \left[ \frac{(k-1)(\tau \ln δ + 1 - εₚ₆ \ln εₚ₆)}{(\tau - 1) + (k-1)\ln δ} \right]. \]  

(5)

In the absence of the preliminary isobaric compression of the working body in the cooler points 6 and 1 in Fig. 2 coincide, (εₚ₆ = v₀/v₁ = 1, v₁ = v₂), εₚ₆ = δ, τ = T₀/T₁ = T₀/T₂, the cycle is transformed into the ideal Stirling cycle with thermal efficiency equal to [4]

\[ η = \frac{[(k-1)(τ - 1)\ln δ]}{[τ(1-1) + (k-1)\ln δ]}. \]  

(6)

Average pressure p₁ of the thermodynamic cycle in Fig.2 equal to the useful work of the cycle received by the unit of the operation volume of the cylinder (i.e. specific work of the cycle) is equal to:

\[ p₁ = \frac{(\sum q_i - \sum q_i')}{(v₂ - v₁)}, \]  

(7)

where (v₁–v₂) – operation volume of the cylinder.

Dimensionless average pressure correlated to the pressure in the point 1 (p₁ = p₀), taking into account equations (3), (4) may be represented as follows:

\[ p₁ = \frac{εₜ(τ \ln δ - εₚ₆ + 1 - \ln εₜ)}{(δ - 1)}. \]  

(8)

In the particular case of the Stirling cycle in the absence of preliminary isobaric compression of the working body and fulfilling the requirements for the parameters εₚ₆, εₜ, δ, τ, tₚ₆ formulated during correlation derivation (6), known result is received:

\[ p₁ = \frac{δ(τ - 1)\ln δ}{(δ - 1)} \]  

(9)

For the numerical assessment of the impact of preliminary isobaric compression of the working body εₚ₆ in the cooler and associated with it pressure ratio increase λ = p₆/p₂ for isothermal efficiency values ηₜ and dimensionless average pressure p₁/p₂, let us consider the extreme case of isobaric compression when isothermal compression by the working piston is not required (absence of the curve 1-2) and due to the decrease of the gas temperature in the cooler from T₅ to T₁ at constant pressure P₅ = P₁ only isobar 6-1 is realized. In this case the density of the gas at point 1 reaches the value of p₁ = (p₀T₅)/T₁. Considered cycle is shown in Fig. 3. In this case, points 1, 2 coincide and fulfill the relations εₜ = v₂/v₁ = 1, δ = εₚ₆.

Results of calculations of the values ηₜ and p₁/p₂ for cases of the Stirling engine without regeneration, discussed in [4], and the modified Stirling engine (with breaker valve, and unidirectional movement of the working body in the recuperative heat exchanger) with the following initial data T₀=T₅ =750K, T₆=T₃ =500K, T₄=T₁=333K, k =1.6, δ = εₚ₆ = 1.5, εₜ = v₂/v₁ = 1, τ = T₄/T₁, τ₀ = T₅/T₆, P₆ =P₁=1bar are shown below.

Cycle 3456 corresponds to classical Stirling engine in Fig. 3. On the bases of the equations (5) - (6) and (8) - (9) taking into account values τ₀ = T₄/T₁=1.5 the following results are received

\[ η_{Stirling} = 0.1406, \quad η_{Recycoler} = 0.1377. \]

The decrease of the efficiency of the modified engine for Δ = 0.29% is related to the replacement of the more effective isothermal compression by the less effective isobaric one. The
dimensionless average pressure in the modified engine is higher than in the classical Stirling engine (value \( r = T_4/T_1 = 2.25 \));
\[ (p_2/p_1)_{\text{Stirling}} = 0.608, (p_2/p_1)_{\text{cooler}} = 0.825 \]
approximately for 35%, which is shown in Fig. 3.

5. **Conclusions**

- The possibility of satisfactory operation of the modified Stirling engine without a regenerator with unidirectional pulsating motion of the working body through the recuperative heat exchangers is demonstrated theoretically.
- It is shown that the implementation of recuperative cooler for the preliminary isobaric cooling of the working body allows increasing the engine power with a small decrease of thermal efficiency.
- The maximal increase of the engine power for the ideal modified engine cycle in comparison compared to the Stirling engine reaches ~ 35% with the less than 0.5% decrease of the thermal coefficient.

6. **Literature**

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