MECHANIC-MATHEMATICAL MODELING OF INDEPENDENT SUSPENSION AS TAKE ACCOUNT ELASTICITY OF SILENT BLOCKS

МЕХАНО-МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ НЕЗАВЫСИМОЙ ПОДВЕСКИ С УЧЕТОМ УПРУГОСТИ РЕЗИНОМЕТАЛЛИЧЕСКИХ ВИБРОИЗОЛЯТОРОВ

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Abstract: *In the paper is investigated dynamic behaviour of one arm independent suspension for a car. For describing the mechanical system is used mathematical model based on vector-matrix algebra. The results from numerical experiments show laws of movement the sprung and unsprung masses, theirs natural frequencies and theirs accelerations.*
***KEYWORDS**: MATHEMATICAL MODELS, VIBRATIONS, NATURAL FREQUENCIES, RIDE COMFORT*

1. Introduction.

Kinematical structure of the vehicle independent suspension with increased speeds of motion have been refined over time and today there are a large number of its variants - one, two, and more arm suspensions. Underlying all these are one arm suspensions. The aim of this work is to investigate the dynamic behavior of a car with one arm suspension as take account the influence of the stiffness of the tires, the main elastic elements (coil springs) and the additional elastic bushings (silent blocks). For this purpose are used the methods of the vector mechanics and numerical experiments are conduct with MATLAB.

2. Three-dimensional mathematical model of arm suspension, describing the smoothness of motion with the methods of vector mechanics.

The behavior of the car is described using the three-dimensional mathematical model. The advantage of this scheme is that it is possible to examine the relocation of the car on the axes Ox, Oy and Oz of the coordinate system located in the center of its mass (i.e. all degrees of freedom) which is a prerequisite for high accuracy in computation process. Scheme of the model is shown in Figure 1.

The system under consideration consists sprung and unsprung masses. The sprung masses include the masses of the elements of the car, passengers and load. In the center of gravity is fixed local coordinate system attached $O_0x_0y_0z_0$. The suspension is implemented as a tire, arm, axle and other components are combined in one element which is fixed to hinged to the sprung masses by elastic bushings (Fig. 2). Each of these elements is fixed to local coordinate system, respectively $O_1x_1y_1z_1$, $O_2x_2y_2z_2$, $O_3x_3y_3z_3$, $O_4x_4y_4z_4$.

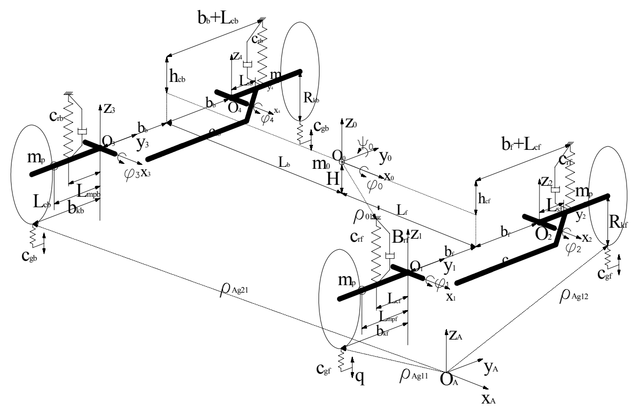

Fig. 1 Kinematic scheme of a car with front and rear transverse arms of suspension

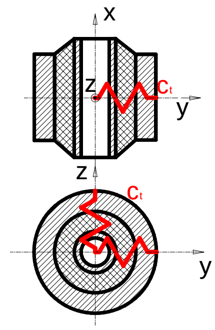

Fig. 2 Drawing of the elastic bushing (silent block)

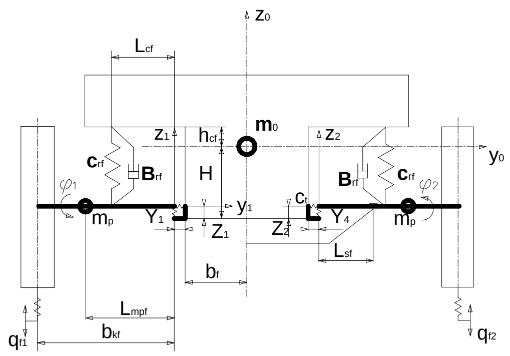

Fig. 3 Scheme of the front axle of a car

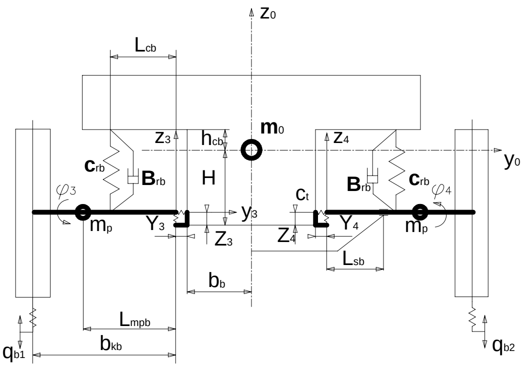

Fig. 4 Scheme of the rear axle of a car

In the equilibrium position the axis of all coordinate systems are parallel. All displacements of local coordinate systems are given to the absolute coordinate system $O_Ax_Ay_Az_A$. In Fig. 3 and 4 are shown the geometrical parameters, respectively front and rear axle of the car.

For system of Fig. 1 make the following assumptions:

-elements of the system are solids;

-anti-roll bars are massless and their stiffness is regarded as equivalent spring connected to the arms at point to a distance L_{sf} of the joint (hinge) of the front axle and L_{sb} of the joint of the rear axle;

-give an account damping and elastic properties of the main elements c_{rf}, c_{rb}, β_{rf}, β_{rb}, respectively, springs and shock absorbers the front and the rear axle, and the elasticity of the tire c_{gf} and c_{gb} the front and the rear axle;

-elastic and damping elements have linear characteristics;

-system is placed in a equilibrium position as the centers of gravity to the wheels lie on a horizontal axis. O_1y_1 axis coincides with the axis O_2y_2, and O_3y_3 axis coincides with O_4y_4.

For generalized coordinate systems are adopted:

- z_0 - linear displacement of the local coordinate system $O_0x_0y_0z_0$ to absolute $O_Ax_Ay_Az_A$ on axis Oz;
- φ_0, ψ_0, θ_0 - angular displacement of the local coordinate system O0x0y0z0 to absolute $O_Ax_Ay_Az_A$ respectively around the axes Ox, Oy and Oz;
- φ_1 - angular displacement around the axis O_1x_1 of the coordinate system $O_1x_1y_1z_1$;
- φ_2 - angular displacement around the axis O_2x_2 of the coordinate system $O_2x_2y_2z_2$;
- φ_3 - angular displacement around the axis O_3x_3 of the coordinate system $O_3x_3y_3z_3$;
- φ_4 - angular displacement around the axis O_4x_4 of the coordinate system $O_4x_4y_4z_4$;
- φ_3 - angular displacement around the axis O_3y_3 of the coordinate system $O_3x_3y_3z_3$;
- φ_4 - angular displacement around the axis O_4y_4 of the coordinate system $O_4x_4y_4z_4$;
- z_1, z_2, z_3, z_4 - vertical displacements of arm points of attachment to the body;
- y_1, y_2, y_3, y_4 - horizontal displacements of arm points of attachment to the body;
- Longitudinal stiffness of the elastic bushings i.e. longitudinal displacement of the arm is not counted.

To find laws of motion in the absolute coordinate system $O_Ax_Ay_Az_A$ is necessary to define the transition matrices of each local coordinate system to the absolute.

-matrix of transition from $O_0x_0y_0z_0$ to $O_Ax_Ay_Az_A$:

$$T_0^A = \begin{bmatrix} \cos\psi_0\cos\theta_0 & \cos\psi_0\sin\theta_0 & -\sin\psi_0 & 0 \\ -\sin\varphi_0\sin\psi_0\cos\theta_0 - \cos\varphi_0\sin\theta_0 & -\sin\varphi_0\sin\psi_0\sin\theta_0 + \cos\varphi_0\cos\theta_0 & -\sin\varphi_0\cos\psi_0 & y_0 \\ \cos\varphi_0\sin\psi_0\cos\theta_0 - \sin\varphi_0\sin\theta_0 & \cos\varphi_0\sin\psi_0\sin\theta_0 + \sin\varphi_0\cos\theta_0 & \cos\varphi_0\cos\psi_0 & z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

x_0 and y_0 are zero because is consider only linear oscillation axis Oz, i.e. only vertically;

matrix of transition from $O_1x_1y_1z_1$, $O_2x_2y_2z_2$, $O_3x_3y_3z_3$, $O_4x_4y_4z_4$, to $O_0x_0y_0z_0$ have a type:

$$T_1^0 = \begin{bmatrix} 1 & 0 & 0 & L_f \\ 0 & \cos\varphi_1 & -\sin\varphi_1 & -b_f - y_1 \\ 0 & \sin\varphi_1 & \cos\varphi_1 & -H + z_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^0 = \begin{bmatrix} 1 & 0 & 0 & L_f \\ 0 & \cos\varphi_2 & -\sin\varphi_2 & b_f + y_2 \\ 0 & \sin\varphi_2 & \cos\varphi_2 & -H + z_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = \begin{bmatrix} 1 & 0 & 0 & -L_b \\ 0 & \cos\varphi_3 & -\sin\varphi_3 & -b_b - y_3 \\ 0 & \sin\varphi_3 & \cos\varphi_3 & -H + z_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4^0 = \begin{bmatrix} 1 & 0 & 0 & -L_b \\ 0 & \cos\varphi_4 & -\sin\varphi_4 & b_b + y_4 \\ 0 & \sin\varphi_4 & \cos\varphi_4 & -H + z_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The components of the angular velocity of sprung masses are set in advance:

$$\omega_{0x}^A = \dot{\varphi}_0$$
$$\omega_{0y}^A = \dot{\psi}_0$$
$$\omega_{0z}^A = \dot{\theta}_0$$

Otherwise however is the issue of determining the angular velocities of the arms where there is transmission and relative motion.

ω_i – the angular velocity of the i-th unit to the absolute coordinate system is equal to:

$$\underline{\omega_i^{0*}} = -T_i^0 \dot{T}_i^{0(T)} = \dot{T}_i^0 T_i^{0(T)}$$

$$\underline{\omega_i^{0*}} = \begin{bmatrix} 0 & -\omega_{iz} & \omega_{iy} \\ \omega_{iz} & 0 & -\omega_{ix} \\ -\omega_{iy} & \omega_{ix} & 0 \end{bmatrix}$$

After multiplying the matrices and simplify the resulting expressions for the components of the angular velocity of the arms and the three axes are obtained:

- front right arm:

$$\omega_{1x}^A = \dot{\varphi}_0 + \dot{\varphi}_1$$
$$\omega_{1y}^A = \dot{\psi}_0$$
$$\omega_{1z}^A = \dot{\theta}_0$$

- front left arm:

$$\omega_{2x}^{A} = \dot{\varphi}_0 + \dot{\varphi}_2$$
$$\omega_{2y}^{A} = \dot{\psi}_0$$
$$\omega_{2z}^{A} = \dot{\theta}_0$$

- rear right arm:

$$\omega_{3x}^{A} = \dot{\varphi}_0 + \dot{\varphi}_3$$
$$\omega_{3y}^{A} = \dot{\psi}_0$$
$$\omega_{3z}^{A} = \dot{\theta}_0$$

- rear left arm:

$$\omega_{4x}^{A} = \dot{\varphi}_0 + \dot{\varphi}_4$$
$$\omega_{4y}^{A} = \dot{\psi}_0$$
$$\omega_{4z}^{A} = \dot{\theta}_0$$

The kinetic energy of the system is:

$$\begin{aligned}
\mathrm{T} = &\frac{1}{2}m_0\dot{z}_0^2 + \frac{1}{2}m_0\dot{y}_0^2 + \frac{1}{2}J_{0x}\dot{\varphi}_0^2 + \frac{1}{2}J_{0y}\dot{\psi}_0^2 + \frac{1}{2}J_{0z}\dot{\theta}_0^2 + \\
&+\frac{1}{2}J_{pxf}(\dot{\varphi}_0 + \dot{\varphi}_1)^2 + \frac{1}{2}J_{pxf}(\dot{\varphi}_0 + \dot{\varphi}_2)^2 + \frac{1}{2}J_{pxb}(\dot{\varphi}_0 + \dot{\varphi}_3)^2 + \\
&+\frac{1}{2}J_{pxb}(\dot{\varphi}_0 + \dot{\varphi}_4)^2 + 2(\frac{1}{2}J_{pyf}\dot{\psi}_0^2) + 2(\frac{1}{2}J_{pyb}\dot{\psi}_0^2) + \\
&+2(\frac{1}{2}J_{rzf}\dot{\theta}_0^2) + 2(\frac{1}{2}J_{rzb}\dot{\theta}_0^2) + \frac{1}{2}m_p(\dot{y}_0 - \dot{y}_1 + H\dot{\varphi}_0 - L_f\dot{\theta}_0)^2 + \\
&+\frac{1}{2}m_p(\dot{z}_0 + \dot{z}_1 - (L_{mpf} + b_f)\dot{\varphi}_0 + L_f\dot{\psi}_0 - L_{mpf}\dot{\varphi}_1)^2 + \\
&+\frac{1}{2}m_p(\dot{y}_0 + \dot{y}_2 + H\dot{\varphi}_0 - L_f\dot{\theta}_0)^2 + \\
&+\frac{1}{2}m_p(\dot{z}_0 + \dot{z}_2 + (L_{mpf} + b_f)\dot{\varphi}_0 + L_f\dot{\psi}_0 + L_{mpf}\dot{\varphi}_2)^2 + \\
&+\frac{1}{2}m_p(\dot{y}_0 - \dot{y}_3 + H\dot{\varphi}_0 + L_b\dot{\theta}_0)^2 + \\
&+\frac{1}{2}m_p(\dot{z}_0 + \dot{z}_3 - (L_{mpb} + b_b)\dot{\varphi}_0 - L_b\dot{\psi}_0 - L_{mpb}\dot{\varphi}_3)^2 + \\
&+\frac{1}{2}m_p(\dot{y}_0 + \dot{y}_4 + H\dot{\varphi}_0 + L_b\dot{\theta}_0)^2 + \\
&+\frac{1}{2}m_p(\dot{z}_0 + \dot{z}_4 + (L_{mpb} + b_b)\dot{\varphi}_0 - L_b\dot{\psi}_0 + L_{mpb}\dot{\varphi}_4)^2
\end{aligned}$$

The potential energy of the system is:

$$\begin{aligned}
\Pi = &\frac{1}{2}c_{rf}(L_{cf}\varphi_1 - z_1)^2 + \frac{1}{2}c_{rf}(-L_{cf}\varphi_2 - z_2)^2 + \frac{1}{2}c_{rb}(L_{cb}\varphi_3 - z_3)^2 \\
&+\frac{1}{2}c_{rb}(-L_{cb}\varphi_4 - z_4)^2 + \frac{1}{2}c_{gy}(y_0 + (H + R_{kf})\varphi_0 - \\
&-L_f\theta_0 + R_{kf}\varphi_1 - y_1 - q_{fy1})^2 + \frac{1}{2}c_{gz}(z_0 - (b_f + b_{kf})\varphi_0 + \\
&+L_f\psi_0 - b_{kf}\varphi_1 + z_1 - q_{f1})^2 + \frac{1}{2}c_{gy}(y_0 + (H + R_{kf})\varphi_0 - \\
&-L_f\theta_0 + R_{kf}\varphi_2 + y_2 - q_{fy2})^2 + \frac{1}{2}c_{gz}(z_0 + (b_f + b_{kf})\varphi_0 + \\
&+L_f\psi_0 + b_{kf}\varphi_2 + z_2 - q_{f2})^2 + \frac{1}{2}c_{gy}(y_0 + (H + R_{kb})\varphi_0 + \\
&+L_b\theta_0 + R_{kb}\varphi_3 - y_3 - q_{by1})^2 + \frac{1}{2}c_{gz}(z_0 - (b_b + b_{kb})\varphi_0 - \\
&-L_b\psi_0 - b_{kb}\varphi_3 + z_3 - q_{b1})^2 + \frac{1}{2}c_{gy}(y_0 + (H + R_{kb})\varphi_0 + \\
&+L_b\theta_0 + R_{kb}\varphi_4 + y_4 - q_{by2})^2 + \frac{1}{2}c_{gz}(z_0 + (b_b + b_{kb})\varphi_0 - \\
&-L_b\psi_0 + b_{kb}\varphi_4 + z_4 - q_{b2})^2 + \frac{1}{2}c_{sf}(-L_{sf}\varphi_1 - L_{sf}\varphi_2 + z_1 - z_2)^2 + \\
&+\frac{1}{2}c_{sb}(-L_{sb}\varphi_3 - L_{sb}\varphi_4 + z_3 - z_4)^2 + \\
&+\frac{1}{2}c_t(-y_1)^2 + \frac{1}{2}c_t(y_2)^2 + \frac{1}{2}c_t(-y_3)^2 + \frac{1}{2}c_t(y_4)^2 + \\
&+\frac{1}{2}c_t(z_1)^2 + \frac{1}{2}c_t(z_2)^2 + \frac{1}{2}c_t(z_3)^2 + \frac{1}{2}c_t(z_4)^2
\end{aligned}$$

The Rayleigh's function is:

$$\begin{aligned}
R = &\frac{1}{2}\beta_{rf}(L_{cf}\dot{\varphi}_1 - z_1)^2 + \frac{1}{2}\beta_{rf}(-L_{cf}\dot{\varphi}_2 - z_2)^2 + \\
&+\frac{1}{2}\beta_{rb}(L_{cb}\dot{\varphi}_3 - z_3)^2 + \frac{1}{2}\beta_{rb}(-L_{cb}\dot{\varphi}_4 - z_4)^2
\end{aligned}$$

After applying Lagrange's equation of 2nd kind:

$$\frac{d}{dt}\left(\frac{\partial \mathrm{T}}{\partial \dot{q}}\right) - \left(\frac{\partial \mathrm{T}}{\partial q}\right) = \left(\frac{\partial \Pi}{\partial q}\right) - \left(\frac{\partial R}{\partial \dot{q}}\right)$$

for equations describing the laws of motion of the system of Figure 1 is valid:

$$[\mathrm{M}]\ddot{q} + [\mathrm{B}]\dot{q} + [C]q = [F]$$

- [M] is the matrix of inertia that is symmetrical with the main diagonal with dimension 17x17 and she has the type of table. 1
- [C] is the matrix of elasticity, which is also symmetric and has 17x17 dimension (Table 2)
- [B] is the matrix of dissipative forces, showing the influence of dampers - with a symmetrical dimension 17x17 and has the following non-zero elements:

$B(6,6) = B(7,7) = \beta rf.L_{cf}^2$
$B(6,10) = B(10,6) = -\beta_{rf}.L_{cf}$
$B(7,11) = B(11,7) = \beta_{rf}.L_{cf}$
$B(8,8) = B(9,9) = \beta_{rb}L_{cb}^2$
$B(8,12) = B(12,8) = -\beta_{rb}L_{cb}$
$B(9,13) = B(13,9) = \beta_{rb}L_{cb}$
$B(10,10) = B(11,11) = \beta_{rf}$
$B(12,12) = B(13,13) = \beta_{rb}$

Table 1

m_0+4m_p	0	0	$2m_pL_f-2m_pL_b$	0	$-m_pLm_{pf}$	m_pL_{mpf}	$-m_pL_{mpb}$	m_pLm_{pb}	m_p	m_p	m_p	m_p	0	0	0	0
0	m_0+4m_p	$4m_pH$	0	$-2m_pL_f+2m_pL_b$	0	0	0	0	0	0	0	0	$-m_p$	m_p	$-m_p$	m_p
0	$4m_pH$	$J_{0x}+2J_{rxf}+2J_{rxb}+4m_pH^2+2m_p(L_{mpf}+b_f)^2+2m_p(L_{mpb}+b_b)^2$	0	$-2m_pHL_f+2m_pHL_b$	$J_{rxf}+m_pL_{mpf}(L_{mpf}+b_f)$	$J_{rxf}+m_pL_{mpf}(L_{mpf}+b_f)$	$J_{rxb}+m_pL_{mpb}(L_{mpb}+b_b)$	$J_{rxb}+m_pL_{mpb}(L_{mpb}+b_b)$	$-m_p(L_{mpf}+b_f)$	$m_p(L_{mpf}+b_f)$	$-m_p(L_{mpb}+b_b)$	$m_p(L_{mpb}+b_b)$	$-m_pH$	m_pH	$-m_pH$	m_pH
$2m_pL_f-2m_pL_b$	0	0	$J_{0y}+2J_{ryf}+2J_{ryb}+2m_pL_f^2+2m_pL_b^2$	0	$-m_pL_fL_{mpf}$	$m_pL_fL_{mpf}$	$m_pL_bL_{mpb}$	$-m_pL_bL_{mpb}$	m_pL_f	m_pL_f	$-m_pL_b$	$-m_pL_b$	0	0	0	0
0	$-2m_pL_f+2m_pL_b$	$-2m_pHL_f+2m_pHL_b$	0	$J_{0z}+2J_{rzf}+2J_{rzb}+2m_pL_f^2+2m_pL_b^2$	0	0	0	0	0	0	0	0	m_pL_f	$-m_pL_f$	$-m_pL_b$	m_pL_b
$-m_pL_{mpf}$	0	$J_{rxf}+m_pL_{mpf}(L_{mpf}+b_f)$	$-m_pL_fL_{mpf}$	0	$J_{rxf}+m_pL_{mpf}^2$	0	0	0	$-m_pL_{mpf}$	0	0	0	0	0	0	0
m_pL_{mpf}	0	$J_{rxf}+m_pL_{mpf}(L_{mpf}+b_f)$	$m_pL_fL_{mpf}$	0	0	$J_{rxf}+m_pL_{mpf}^2$	0	0	0	m_pL_{mpf}	0	0	0	0	0	0
$-m_pL_{mpb}$	0	$J_{rxb}+m_pL_{mpb}(L_{mpb}+b_b)$	$m_pL_bL_{mpb}$	0	0	0	$J_{rxb}+m_pL_{mpb}^2$	0	0	0	$-m_pL_{mpb}$	0	0	0	0	0
m_pL_{mpb}	0	$J_{rxb}+m_pL_{mpb}(L_{mpb}+b_b)$	$-m_pL_bL_{mpb}$	0	0	0	0	$J_{rxb}+m_pL_{mpb}^2$	0	0	0	m_pL_{mpb}	0	0	0	0
m_p	0	$-m_p(L_{mpf}+b_f)$	m_pL_f	0	$-m_pL_{mpf}$	0	0	0	m_p	0	0	0	0	0	0	0
m_p	0	$m_p(L_{mpf}+b_f)$	m_pL_f	0	0	m_pL_{mpf}	0	0	0	m_p	0	0	0	0	0	0
m_p	0	$-m_p(L_{mpb}+b_b)$	$-m_pL_b$	0	0	0	$-m_pL_{mpb}$	0	0	0	m_p	0	0	0	0	0
m_p	0	$m_p(L_{mpb}+b_b)$	$-m_pL_b$	0	0	0	0	m_pL_{mpb}	0	0	0	m_p	0	0	0	0
0	$-m_p$	$-m_pH$	0	m_pL_f	0	0	0	0	0	0	0	0	m_p	0	0	0
0	m_p	m_pH	0	$-m_pL_f$	0	0	0	0	0	0	0	0	0	m_p	0	0
0	$-m_p$	$-m_pH$	0	m_pL_b	0	0	0	0	0	0	0	0	0	0	m_p	0
0	m_p	m_pH	0	m_pL_b	0	0	0	0	0	0	0	0	0	0	0	m_p

Table 2

$4c_{gz}$	0	0	$2c_{gz}L_f-2c_{gz}L_b$	0	$-c_{gz}b_{kf}$	$c_{gz}b_{kf}$	$-c_{gz}b_{kb}$	$c_{gz}b_{kb}$	c_{gz}	c_{gz}	c_{gz}	c_{gz}	0	0	0	0
0	$4c_{gy}$	$2c_{gy}(H+R_{kf})+2c_{gy}(H+R_{kb})$	0	$-2c_{gy}L_f+2c_{gy}L_b$	$c_{gy}R_{kf}$	$c_{gy}R_{kf}$	$c_{gy}R_{kb}$	$c_{gy}R_{kb}$	0	0	0	0	$-c_{gy}$	c_{gy}	$-c_{gy}$	c_{gy}
0	$2c_{gy}(H+R_{kf})+2c_{gy}(H+R_{kb})$	$2c_{gy}(H+R_{kf})^2+2c_{gy}(H+R_{kb})^2+2c_{gz}(b_f+b_{kf})^2+2c_{gz}(b_b+b_{kb})^2$	0	$-2c_{gy}L_f(H+R_{kf})+2c_{gy}L_b(H+R_{kb})$	$c_{gy}R_{kf}(H+R_{kf})+c_{gz}b_{kf}(b_f+b_{kf})$	$c_{gy}R_{kf}(H+R_{kf})+c_{gz}b_{kf}(b_f+b_{kf})$	$c_{gy}R_{kb}(H+R_{kb})+c_{gz}b_{kb}(b_b+b_{kb})$	$c_{gy}R_{kb}(H+R_{kb})+c_{gz}b_{kb}(b_b+b_{kb})$	$-c_{gz}(b_f+b_{kf})$	$c_{gz}(b_f+b_{kf})$	$-c_{gz}(b_b+b_{kb})$	$c_{gz}(b_b+b_{kb})$	$-c_{gy}(H+R_{kf})$	$c_{gy}(H+R_{kf})$	$-c_{gy}(H+R_{kb})$	$c_{gy}(H+R_{kb})$
$2c_{gz}L_f-2c_{gz}L_b$	0	0	$2c_{gz}L_f^2+2c_{gz}L_b^2$	0	$-c_{gz}L_fb_{kf}$	$c_{gz}L_fb_{kf}$	$c_{gz}L_bb_{kb}$	$-c_{gz}L_bb_{kb}$	$c_{gz}L_f$	$c_{gz}L_f$	$-c_{gz}L_b$	$-c_{gz}L_b$	0	0	0	0
0	$-2c_{gy}L_f+2c_{gy}L_b$	$-2c_{gy}L_f(H+R_{kf})+2c_{gy}L_b(H+R_{kb})$	0	$-2c_{gy}L_f^2+2c_{gy}L_b^2$	$-c_{gy}L_fR_{kf}$	$-c_{gy}L_fR_{kf}$	$c_{gy}L_bR_{kb}$	$c_{gy}L_bR_{kb}$	0	0	0	0	$c_{gy}L_f$	$-c_{gy}L_f$	$-c_{gy}L_b$	$c_{gy}L_b$
$-c_{gz}b_{kf}$	$c_{gy}R_{kf}$	$c_{gy}R_{kf}(H+R_{kf})+c_{gz}b_{kf}(b_f+b_{kf})$	$-c_{gz}L_fb_{kf}$	$-c_{gy}L_fR_{kf}$	$c_{rf}L_{cf}^2+c_{gy}R_{kf}^2+c_{gz}b_{kf}^2+c_{sf}L_{sf}^2$	$c_{sf}L_{sf}^2$	0	0	$-c_{rf}L_{cf}-c_{gz}b_{kf}-c_{sf}L_{sf}$	$c_{sf}L_{sf}$	0	0	$-c_{gy}R_{kf}$	0	0	0
$c_{gz}b_{kf}$	$c_{gy}R_{kf}$	$c_{gy}R_{kf}(H+R_{kf})+c_{gz}b_{kf}(b_f+b_{kf})$	$c_{gz}L_fb_{kf}$	$-c_{gy}L_fR_{kf}$	$c_{sf}L_{sf}^2$	$c_{rf}L_{cf}^2+c_{gy}R_{kf}^2+c_{gz}b_{kf}^2+c_{sf}L_{sf}^2$	0	0	$-c_{sf}L_{sf}$	$c_{rf}L_{cf}+c_{gz}b_{kf}+c_{sf}L_{sf}$	0	0	0	$c_{gy}R_{kf}$	0	0
$-c_{gz}b_{kb}$	$c_{gy}R_{kb}$	$c_{gy}R_{kb}(H+R_{kb})+c_{gz}b_{kb}(b_b+b_{kb})$	$c_{gz}L_bb_{kb}$	$c_{gy}L_bR_{kb}$	0	0	$c_{rb}L_{cb}^2+c_{gy}R_{kb}^2+c_{gz}b_{kb}^2+c_{sb}L_{sb}^2$	$c_{sb}L_{sb}^2$	0	0	$-c_{rb}L_{cb}-c_{gz}b_{kb}-c_{sb}L_{sb}$	$c_{sb}L_{sb}$	0	0	$-c_{gy}R_{kb}$	0
$c_{gz}b_{kb}$	$c_{gy}R_{kb}$	$c_{gy}R_{kb}(H+R_{kb})+c_{gz}b_{kb}(b_b+b_{kb})$	$-c_{gz}L_bb_{kb}$	$c_{gy}L_bR_{kb}$	0	0	$c_{sb}L_{sb}^2$	$c_{rb}L_{cb}^2+c_{gy}R_{kb}^2+c_{gz}b_{kb}^2+c_{sb}L_{sb}^2$	0	0	$-c_{sb}L_{sb}$	$c_{rb}L_{cb}+c_{gz}b_{kb}+c_{sb}L_{sb}$	0	0	0	$c_{gy}R_{kb}$
c_{gz}	0	$-c_{gz}(b_f+b_{kf})$	$c_{gz}L_f$	0	$-c_{rf}L_{cf}-c_{gz}b_{kf}-c_{sf}L_{sf}$	$-c_{sf}L_{sf}$	0	0	$c_{rf}+c_{gz}+c_{sf}+c_t$	$-c_{sf}$	0	0	0	0	0	0
c_{gz}	0	$c_{gz}(b_f+b_{kf})$	$c_{gz}L_f$	0	$c_{sf}L_{sf}$	$c_{rf}L_{cf}+c_{gz}b_{kf}+c_{sf}L_{sf}$	0	0	$-c_{sf}$	$c_{rf}+c_{gz}+c_{sf}+c_t$	0	0	0	0	0	0
c_{gz}	0	$-c_{gz}(b_b+b_{kb})$	$-c_{gz}L_b$	0	0	0	$-c_{rb}L_{cb}-c_{gz}b_{kb}-c_{sb}L_{sb}$	$-c_{sb}L_{sb}$	0	0	$c_{rb}+c_{gz}+c_{sb}+c_t$	$-c_{sb}$	0	0	0	0
c_{gz}	0	$c_{gz}(b_b+b_{kb})$	$-c_{gz}L_b$	0	0	0	$c_{sb}L_{sb}$	$c_{rb}L_{cb}+c_{gz}b_{kb}+c_{sb}L_{sb}$	0	0	$-c_{sb}$	$c_{rb}+c_{gz}+c_{sb}+c_t$	0	0	0	0
0	$-c_{gy}$	$-c_{gy}(H+R_{kf})$	0	$c_{gy}L_f$	$-c_{gy}R_{kf}$	0	0	0	0	0	0	0	$c_{gy}+c_t$	0	0	0
0	c_{gy}	$c_{gy}(H+R_{kf})$	0	$-c_{gy}L_f$	0	$c_{gy}R_{kf}$	0	0	0	0	0	0	0	$c_{gy}+c_t$	0	0
0	$-c_{gy}$	$-c_{gy}(H+R_{kb})$	0	$-c_{gy}L_b$	0	0	$-c_{gy}R_{kb}$	0	0	0	0	0	0	0	$c_{gy}+c_t$	0
0	c_{gy}	$c_{gy}(H+R_{kb})$	0	$c_{gy}L_b$	0	0	0	$c_{gy}R_{kb}$	0	0	0	0	0	0	0	$c_{gy}+c_t$

Generalized coordinates and their derivatives are:

$$\{q\}=\begin{bmatrix} z_0 \\ y_0 \\ \varphi_0 \\ \psi_0 \\ \theta_0 \\ \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \varphi_4 \\ z_1 \\ z_2 \\ z_3 \\ z_4 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} \quad \{\dot{q}\}=\begin{bmatrix} \dot{z}_0 \\ \dot{y}_0 \\ \dot{\varphi}_0 \\ \dot{\psi}_0 \\ \dot{\theta}_0 \\ \dot{\varphi}_1 \\ \dot{\varphi}_2 \\ \dot{\varphi}_3 \\ \dot{\varphi}_4 \\ \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \\ \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \\ \dot{y}_4 \end{bmatrix} \quad \{\ddot{q}\}=\begin{bmatrix} \ddot{z}_0 \\ \ddot{y}_0 \\ \ddot{\varphi}_0 \\ \ddot{\psi}_0 \\ \ddot{\theta}_0 \\ \ddot{\varphi}_1 \\ \ddot{\varphi}_2 \\ \ddot{\psi}_3 \\ \ddot{\psi}_4 \\ \ddot{z}_1 \\ \ddot{z}_2 \\ \ddot{z}_3 \\ \ddot{z}_4 \\ \ddot{y}_1 \\ \ddot{y}_2 \\ \ddot{y}_3 \\ \ddot{y}_4 \end{bmatrix}$$

To obtain natural frequencies of the system, equations are presented in the normal Cauchy form:

$$y + Ly = 0$$

Where L is:

$$L=\begin{bmatrix} M^{-1}B & M^{-1}C \\ I & O \end{bmatrix}$$

The output parameters of the system are vibration displacement, vibration velocity, vibration acceleration and they are obtained from the equations:

$$y + Ly = Y$$

Where Y is:

$$Y=\begin{bmatrix} M^{-1}F(t) \\ O \end{bmatrix}$$

After integration of the system using the method of Runge - Kutta receive all decisions in a given time interval.

2. Numerical investigations.

The main parameters and their numerical values (Table 3) are not measured by the authors and are taken from cited literary sources :

Table 3

№	Parameter	Symbol	Value
1.	Sprung masses	m_0	1400 kg
2.	Unsprung masses	m_p	30 kg
3.	Moment of inertia of the sprung masses around longitudinal axis (x-axis)	J_{0x}	550 $kg.m^2$
4.	Moment of inertia of the sprung masses around transverse axis (y-axis)	J_{0y}	2000 $kg.m^2$
5.	Moment of inertia of the unsprung masses on the front axle around x-axis	J_{pxf}	5 $kg.m^2$
6.	Moment of inertia of the unsprung masses on the rear axle around x-axis	J_{pxb}	2 $kg.m^2$
7.	Moment of inertia of the unsprung masses on the front axle around y-axis	J_{pyf}	2 $kg.m^2$
8.	Moment of inertia of the unsprung masses on the rear axle around y-axis	J_{pyb}	5 $kg.m^2$
9.	Vertical co-ordinate of the center of gravity of the unsprung masses in relation to joint of the arms	H	0,4 m
10.	Horizontal co-ordinate of the center of gravity of the unsprung masses in relation to joint of the front arms	b_f	0,4 m
11.	Horizontal co-ordinate of the center of gravity of the unsprung masses in relation to joint of the rear arms	b_b	0,6 m
12.	Distance from the center of gravity to the front axle	L_f	1,1 m
13.	Distance from the center of gravity to the rear axle	L_b	1,5 m
14.	Length of the front arm	b_{kf}	0,42 m
15.	Length of the rear arm	L_{kb}	0,42 m
16.	Stiffness coefficient of the elastic bushings	c_t	1000000 N/m
17.	Distance from the center of gravity of the front(f) and the rear(b) arm to the respective joint	L_{mp}	0,4 m
18.	Distance from fixing point of the front(f) and the rear(b) main elastic element to the respective joint	L_c	0,3 m
19.	Distance from fixing point of the front(f) and the rear(b) anti-roll bar to the respective joint	L_s	0,28 m
20.	Radius of the front(f) and the rear(b) wheels	R_k	0,26 m
21.	Stiffness coefficient of the main elastic elements of the front axle	c_{rf}	25000 N/m
22.	Stiffness coefficient of the main elastic elements of the rear axle	c_{rb}	25000 N/m
23.	Stiffness coefficient of the tyre	c_{gz}	125000 N/m
24.	Stiffness coefficient of the anti-roll bars of the front(f) and the rear(b) axle	c_s	20000 N/m
25.	Damping coefficient of the front(f) and the rear(b) shock absorbers	β_r	1900 N.s/m

Natural frequencies of the system:

- 1.47 Hz - frequency of linear oscillations of sprung masses on z-axis;
- 1.53 Hz - frequency of linear oscillations of sprung masses on y-axis;
- 2.09 Hz - frequency of angular oscillation of the sprung masses around x-axis;
- 1.29 Hz - frequency of angular oscillation of the sprung masses

around y-axis;
- 1.88 Hz - frequency of angular oscillation of the sprung masses around z-axis;
- 8.41 Hz - angular frequency of the front arms around x-axis;
- 8.80 Hz - angular frequency of the rear arms around x-axis;
- 42 and 30 Hz - frequencies of linear oscillations of the arms at the points of attachment to the body on z-axis, and y-axis.

Disturbing actions in the system are sinusoidal and are attached in the center of the contact patch of the tire with the road. They have the following form:

$$q = q_0(1-\cos(vt))$$

q0 = 0,02 m - height of the amplitude of roughness;
υ - circular frequency of the disturbing action:

$$\upsilon = \frac{2\pi.v}{S}, rad/s$$

The frequency of the disturbing action expressed in hertz:

$$\upsilon = \frac{1}{2\pi}\frac{2\pi.v}{S}, Hz$$

v - velocity of the car, m / s;
S - wavelength, m.

As the maximum accelerations are important, the investigated of the behavior of individual elements of the system was conducted at a frequency effects similar to their natural frequencies. The results obtained for some of the accelerations are shown in figures below:

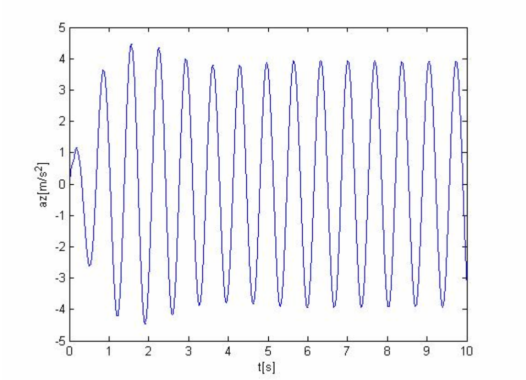

Fig.5 Linear acceleration of the sprung masses on z-axis

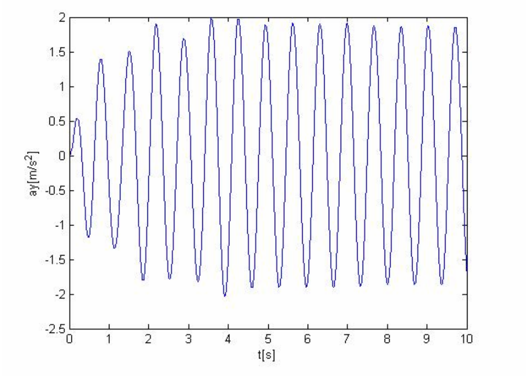

Fig.6 Linear acceleration of the sprung masses on y-axis

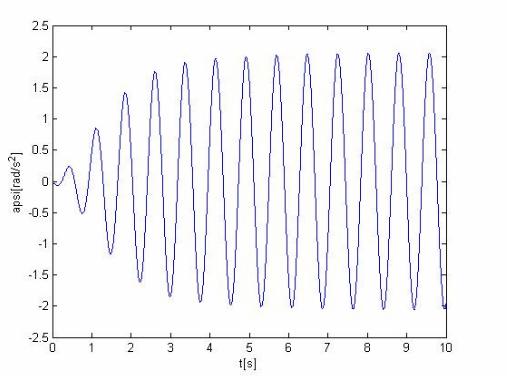

Fig.7 Angular acceleration of the sprung masses around y-axis

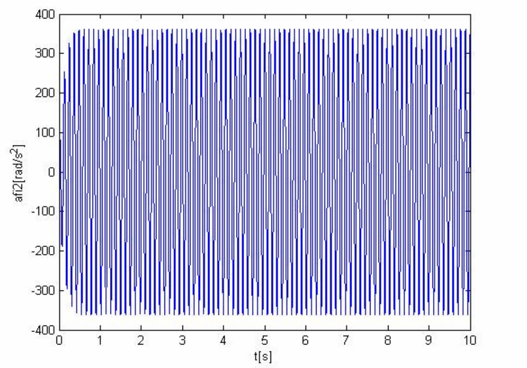

Fig. 8 Angular acceleration of the front left arm

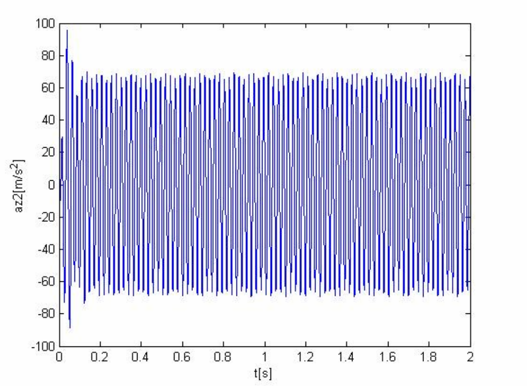

Fig. 9 Linear acceleration on z-axis of the front left arm at the point of attachment to the body

4. Conclusion

Created three-dimensional mathematical model with sufficient accuracy for engineering practice can be using for investigation of dynamic behavior of cars with independent suspensions. The results obtained from this numerical experiment can be considered more reliable because the model takes into account the influence of silent blocks.

5. Acknowledgement

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